COMPLEX INTERVAL ARITHMETICS

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Abstract: *the interval we call a closed segment of the real axis, and interval uncertainty is a state of incomplete knowledge about the value of interest to us, when only its belonging to a certain interval is known, when we can indicate only the limits of possible values of this quantity. Accordingly, interval analysis is a branch of mathematical knowledge that investigates problems with interval uncertainties and methods for solving them. It is possible to give a more detailed definition. Each scientific discipline is characterized, as is well known, by its individual subject and its own specific method. Interval analysis is a branch of mathematics.*
Keywords: *interval, interval analysis, complex number, resistance.*

КОМПЛЕКСНЫЕ ИНТЕРВАЛЬНЫЕ АРИФМЕТИКИ

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Аннотация: *интервалом мы называем замкнутый отрезок вещественной оси, а интервальная неопределённость - это состояние неполного знания об интересующей нас величине, когда известна лишь её принадлежность некоторому интервалу, когда мы можем указать лишь границы возможных значений этой величины. Соответственно, интервальный анализ - это отрасль математического знания, исследующая задачи с интервальными неопределённостями и методы их решения. Можно дать и более развёрнутое определение. Каждая научная дисциплина характеризуется, как известно, своим отдельным предметом и собственным специфическим методом. Интервальный анализ - это раздел математики.*
Ключевые слова: *интервал, интервальный анализ, комплексное число, сопротивление.*

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Interval analysis and its specific methods are therefore of the highest value in problems where uncertainties and ambiguities arise from the very beginning and are an integral part of the formulation of the problem. Although this in no way excludes other fruitful applications of interval analysis, in particular, in problems that are formulated in general without invoking the concept of an interval. The interval $[a,b]$ of the real axis R we call the set of all numbers located between a и b given numbers and, including themselves,

$$[a,b] := \{x \in R | a \le x \le b\}.$$

In this case, a and b they are called the ends of the interval $[a,b]$, left and right, respectively, and the set of all intervals is denoted by the symbol IR. In contrast to intervals and interval values, we will call point values quantities whose values are individual points — the real axis, plane, or, more generally, of some space. The basic idea of interval analysis is to construct a calculus of intervals, learn how to operate with it in the same way as with ordinary numbers, and then use the constructed technique to solve various problems, where the intervals are found in the form of data. The important tools of interval analysis are the so-called interval arithmetic - algebraic systems, which formalize operations on intervals as integral objects. In contrast to the one-dimensional R real axis, the set of complex numbers C is already “two-dimensional”: each element of it needs to describe two numbers, which can be real and imaginary parts, or a module and argument of a complex number. Accordingly, the intervals on the complex plane can be determined in several ways [1].

The most popular complex intervals are rectangles and circles of the complex plane . For intervals $a, b \in IR$, rectangles and circles $a+ib$ are called a complex interval set complex

$\{z = a+ib \in C | a \in a, b \in b\}$.

Circular complex interval $\langle c, r\rangle$, where $c \in C$ и $r \in R_+$, called the complex plane set

$\{z \in C \| z-c| \le r\}$.

The set of all rectangular complex intervals will be denoted IC_{rect}, by the set of circular complex intervals $-IC_{circ}$. If a particular type of arithmetic is unimportant or it is clear from the context of what exactly we are talking about, then for complex interval arithmetic the general designation will be used IC.

For a rectangular complex interval $z = a+ib$

$|z| = \sqrt{|a|^2 + |b|^2}$,

and for a circular complex interval $z = \langle c, r\rangle$

$|z| = |c| + r$.

We agree for the interval $a \in IR$ to mean a^2 the interval extension of the squaring function $x \to x^2$, $a^2 = \{a^2 | a \in a\}$. The constructive definition of this operation is

$$a^2 = \begin{cases} \left[\underline{a}^2, \overline{a}^2\right], \textit{если} \quad a \ge 0 \\ \left[0, \max\left\{\underline{a}^2, \overline{a}^2\right\}\right], \textit{если} \quad 0 \in a \\ \left[\underline{a}^2, \overline{a}^2\right], \textit{если} \quad a \le 0. \end{cases}$$

Then the arithmetic operations over rectangular complex intervals are defined as follows:

$$(a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2),$$
$$(a_1 + ib_1) - (a_2 + ib_2) = (a_1 - a_2) + i(b_1 - b_2),$$
$$(a_1 + ib_1) \cdot (a_2 + ib_2) = (a_1 a_2 - b_1 b_2) + i(a_1 b_2 + a_2 b_1),$$
$$(a_1 + ib_1)/(a_2 + ib_2) = \frac{1}{a_2^2 + b_2^2}((a_1 a_2 + b_1 b_2) + i(a_2 b_1 - a_1 b_2)),$$

moreover, the division is feasible with $0 \notin (a_2 + ib_2)$. The division generally requires special precautions, because by virtue of the repeated entry a_2 and b_2 into the above formula, the rectangular complex interval obtained with its help is in the general case much wider than the exact range of values of the quotient $\{z_1/z_2 | z_1 \in (a_1 + ib_1), z_2 \in (a_2 + ib_2)\}$. For example, if the intervals $(a_1 + ib_1)$ and $(a_2 + ib_2)$ turn out to be real when $b_1 = b_2 = 0$, the result of performing a complex rectangular division $(a_1 + ib_1)/(a_2 + ib_2)$ interval arithmetic IR, a_1/a_2. Often in practice, the definition of division is more preferable. Arithmetic operations on circular complex intervals are defined as follows:

$$\langle c_1, r_1\rangle + 2\langle c_2, r_2\rangle = \langle c_1 + c_2, r_1 + r_2\rangle,$$
$$\langle c_1, r_1\rangle - \langle c_2, r_2\rangle = \langle c_1 - c_2, r_1 + r_2\rangle,$$
$$\langle c_1, r_1\rangle \cdot \langle c_2, r_2\rangle = \langle c_1 c_2, |c_1| r_2 + |c_2| r_1 + r_1 r_2\rangle,$$
$$\frac{1}{\langle c_1, r_1\rangle} = \left\langle \frac{c^*}{|c|^2 - r^2}, \frac{r}{|c|^2 - r^2} \right\rangle,$$
$$\langle c_1, r_1\rangle / \langle c_2, r_2\rangle = \langle c_1, r_1\rangle \cdot \frac{1}{\langle c_2, r_2\rangle},$$

Where $c^* = \mathrm{Re}\,c - \mathrm{Im}\,c$ -complex number, c.

Show that

$\{z_1 z_2 | z_1 \in \langle c_1, r_1 \rangle, z_2 \in \langle c_2, r_2 \rangle\} \subseteq \langle c_1, r_1 \rangle \cdot \langle c_2, r_2 \rangle.$

Really,

$$|z_1 z_2 - c_1 c_2| = |c_1(z_2 - c_2) + c_2(z_1 - c_1) + (z_1 - c_1)(z_2 - c_2)|$$

$$\le |c_1||z_2 - c_2| + |c_2||z_1 - c_1| + |z_1 - c_1||z_2 - c_2| \le |c_1| r_2 + |c_2| r_1 + r_1 r_2.$$

But how much the left and right parts of the inclusion differ from each other. The interval product $\langle 1,1 \rangle \cdot \langle -1+i,1 \rangle$ - circle of the complex plane centered in $(-1+i)$ - in comparison with the multitude of multiplication results. Algebraic properties of complex interval arithmetic:

$u + v = v + u,$ -commutativity of addition,

$(u+v)+w = u+(v+w),$ -associativity of addition,

$u \cdot v = v \cdot u,$ -commutativity of multiplication,

$u(v+w) \subseteq uv + uw,$ - sudistributiveness.

In this case, as in the real case, $u(v+w) = uv + uw$ for any $u \in C$ and $v, w \in IC.$

Note that multiplication in rectangular complex interval arithmetic IC_{rect} does not possess associativity. For example, for $u = ([1,2] + i),$ $v = (1+i)$ and $w = (1+i)$ we have

$(uv)w = ([0,1] + i[2,3]) \cdot (1+i) = [-3,-1] + i[2,4],$

$u(vw) = ([1,2] + i) \cdot 2i = -2 + i[2,4].$

The reason for this loss lies in the absence of the distributivity of multiplication by addition in real interval arithmetic IR. But in circular complex interval arithmetic, the associativity of multiplication is performed:

$(uv)w = u(vw)$ for any $u, v, w \in IC_{circ}.$

Nowadays, complex numbers make it easier to solve physical, chemical, technical, and other issues.

References / Список литературы

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