IDENTIFICATION OF MODAL PARAMETERS USING KALMAN FILTER

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ОПРЕДЕЛЕНИЕ МОДАЛЬНЫХ ПАРАМЕТРОВ С ИСПОЛЬЗОВАНИЕМ ФИЛЬТРА КАЛМАНА

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Аннотация: *применение системы идентификации для вибрационных структур состоит в определении модальных параметров (собственных частот, демпфирующих коэффициентав и формы колебаний) из данных о вибрации. Для динамических характеристик, теория управления, основанная на передаточной функции представления, называется классической теорией управления, в отличие от методологии линейной теории систем на основе анализа временных рядов с помощью фильтра Калмана, и представление пространства состояний называется современной теорией управления. В этой статье мы рассмотрим методику идентификации модальных параметров динамической системы структур с помощью фильтра Калмана, который является мощным средством современной теории управления. Эффективность этого метода идентификации структуры оценивается через моделируемый анализ нескольких степеней свободы вибрации.*

Abstract: *the application of system identification to vibrating structures consists of identifying the modal parameters (eigenfrequencies, damping ratios and mode shapes) from vibration data. For the dynamic characteristics, the control theory based on the transfer function representation is called the classical control theory, in contrast with, the methodology of the linear system theory based on the analy- sis of the time series by kalman filter and the representation of the state space is called modern control theory. In this paper, we consider the methodology of identifying the mode parameters of the dynamic system of structures by using the Kalman filter, which is a powerful means of modern control theory. The effectiveness of this structure identification method is evaluated through simulated analysis of multi - degrees of freedom vibration sytem.*

Ключевые слова: *фильтр Калмана, модальный анализ, собственная частота, формы колебаний, коэффициент демпфирования*

Keywords: *kalman filter, modal analysis, normal frequence, mode shap, damping ratio.*

1. State-space model

The equations of motion for an n_d degrees-of-freedom (DOF) linear, time invariant, viscously damped system subjected to external excitation are expressed as

$$\mathrm{M}\ddot{\mathrm{z}}(t) + C_\zeta \dot{z}(t) + \mathrm{K}\mathrm{z}(t) = Ju(t) \qquad (1)$$

Where $\mathrm{M}, C_\zeta, K \in R^{n_d \times n_d}$ are the mass, damping and stiffness matrices, respectively; $\mathrm{J} \in R^{n_d \times n_d}$ is the excitation influence matrix that relates the n_i-dimensional input vector $\mathrm{u}(t)$ to the n_d-dimensional response vector; $\mathrm{z}(t)$ is the n_d-dimensional displacement response vector; dot denotes taking derivatives with respect to time.

By defining the state vector $x(t) = [z(t) \;\; \dot{z}(t)]^T$, equation (1) can be converted into the continuous state space form

$$\dot{x}(t) = A_c\, x(t) + B_c u(t) \qquad (2)$$

Where

$$A_c = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C_\zeta \end{bmatrix},\; B_c = \begin{bmatrix} 0 \\ M^{-1}J \end{bmatrix} \qquad (3)$$

In practice, only a limited number of measurements are available; therefore, the dimension of the measurement output is less than or equal to the total number of degrees of freedom.

The n_0-dimensional output vector $y(t)$ can be expressed as

$$y(t) = C_d(t)\, z(t) + C_V\, \dot{z}(t) + C_a\, \ddot{z}(t) \qquad (4)$$

Where $C_d, C_V, C_a \in R^{n_0 \times n_d}$ are the measurement location matrices corresponding to the displacement, velocity and acceleration responses of the structural system.

We can rewrite the output vector into the continuous state space form,

$$y(t) = C_c\, x(t) + D_c\, u(t) \qquad (5)$$

Where

$$C_c = \begin{bmatrix} C_d - C_a M^{-1} K & C_V - C_a M^{-1} C_\zeta \end{bmatrix},\;\; D_c = C_a M^{-1} J \qquad (6)$$

In practical application, accelerations are often used commonly, so in this work, only accelerations are considered. Therefore, the C_c of equations (6) is as simple as follows.

$$C_c = C_a \begin{bmatrix} -M^{-1}K & -M^{-1}C_\zeta \end{bmatrix} \qquad (7)$$

Equations 2 and 5 define the state space equation in continuous time:

$$\dot{x}(t) = A_c\, x(t) + B_c\, u(t) \qquad (8a)$$

$$y(t) = C_c\, x(t) + D_c\, u(t) \qquad (8b)$$

Equation (8a) is known as the State Equation and equation (8b) is known as the Observation Equation. But measurements are taken in discrete time instants, so equations must be expressed in discrete time too.

Typical for the sampling of a continuous-time equation is a Zero-Order Hold assumption, which means that the input is piecewise constant over the sampling period, that is

$$\forall t \in [t_k, t_{k+1}) = [k\Delta t, (k+1)\Delta t) \Rightarrow x(t) = x(t_k) = x_k,$$
$$u(t) = u(t_k) = u_k, y(t) = y(t_k) = y_k \qquad (9)$$

Under this assumption, the continuous time state-space model (8a) and (8b) is converted to the discrete time state-space model:

$$x_{k+1} = A\, x_k + B\, u_k \qquad (10a)$$

$$y_k = C\, x_k + D\, u_k \qquad (10b)$$

Where x_k is the discrete time state vector containing the sampled displacements and velocities; u_k and y_k are the sampled input and output; A is the discrete state matrix; B is the discrete input matrix; C is the discrete output matrix; D is the discrete direct transmission matrix. They are related to their continuous-time counterparts as ([2])

$$A = e^{A_c \Delta t}, \quad B = (A - I) A_c^{-1} B_c \qquad (11)$$

$$C = C_c, \quad D = D_c \qquad (12)$$

In system identification, system response disturbance might be caused by different phenomena. The most obvious one is noise generated by the sensors, or noise arising from round off errors during A/D conversion.

It is necessary to extend the state space model (10a) and (10b) including stochastic components, so stochastic state space model is obtained.

$$x_{k+1} = A\,x_k + B\,u_k + w_k \qquad (13a)$$

$$y_k = C\,x_k + D\,u_k + v_k \qquad (13b)$$

Where $\mathrm{w}_k \in \mathrm{R}^n$ is the process noise due to disturbances and modeling inaccuracies; $\mathrm{v}_k \in \mathrm{R}^{n_0}$ is the measurement noise due to sensor inaccuracy.

We assume they are both independent and identically distributed, zero-mean normal vectors.

$$\mathrm{w}_k \sim N(0,Q) \qquad v_k \sim N(0,Q) \qquad (14)$$

2. The Kalman filter

Due to the noise present in the stochastic state space Equations (13), it is only possible to predict the response in term of probability. For state space systems, this prediction is accomplished by the construction of the associated Kalman filter.

For the state space model specified in (13) with initial conditions $\mathrm{x}_0^0 = \mu_0$ and $P_0^0 = \Sigma_0$, for k = 1,2,...,N

$$\mathrm{x}_k^{k-1} = \mathrm{A}\,\mathrm{x}_{k-1}^{k-1} \qquad (15)$$

$$P_k^{k-1} = \mathrm{A}\,\mathrm{P}_{k-1}^{k-1}\,A^T + Q \qquad (16)$$

$$\text{With} \quad \mathrm{x}_k^k = \mathrm{x}_k^{k-1} + K_k\,\varepsilon_k \qquad (17)$$

$$P_k^k = (I - K_k\,C)\mathrm{P}_k^{k-1} \qquad (18)$$

$$\text{Wher} \quad K_k = P_k^{k-1}\,C^T\,\Sigma_k^{-1} \qquad (19)$$

$$\varepsilon_k = y_k - E[y_k|y_{k-1}] = y_k - C\,x_k^{k-1} \qquad (20)$$

$$\Sigma_k = Var(\varepsilon_k) = Var\left[C\left(x_k - x_k^{k-1}\right) + v_k\right] = C\,P_k^{k-1}\,C^T + R \qquad (21)$$

K_k is called the Kalman gain and ε_k are the innovations.

Under stationary conditions,

$$\lim_{k\to\infty} P_k^{k-1} = P > 0 \qquad (22)$$

$$\mathrm{P} = \mathrm{A}\,\mathrm{P}\,A^T + Q - A\,P\,C^T(C\,P\,C^T + R)^{-1}\,(A\,P\,C^T)^T \qquad (23)$$

$$\mathrm{K} = \mathrm{A}\,\mathrm{P}\,C^T(C\,P\,C^T + R)^{-1} \qquad (24)$$

3. System identification and modal analysis in a state-space model

The natural frequencies and modal damping ratios can be retrieved from the eigenvalues of A, and the mode shapes can be evaluated using the corresponding eigenvectors and the output matrix C. The eigenvalues of A come in complex conjugate pairs and each pair represents one physical vibration mode.

Assuming low and proportional damping, the second order modes are uncoupled and the jth eigenvalue of A has the form

$$\lambda_j = exp\left(\left(-\zeta_j\,\omega_j\ \pm i\,\omega_j\,\sqrt{i - \xi_j^2}\right)\Delta t\right) \qquad (25)$$

Where ω_j are the natural frequencies, ξ_j are damping ratios, and Δt is the time step.

Natural frequencies ω_j and the damping ratios ξ_j are given by

$$\omega_j = \frac{|ln(\lambda_j)|}{\Delta t}, \qquad \xi_j = \frac{-Real[ln(\lambda_j)]}{\omega_j\,\Delta t} \qquad (26)$$

The jth mode shape $\phi_j \in R^{n_0}$ evaluated at sensor locations can be obtained using the following expression:

$$\phi_j = \boldsymbol{C}\,\psi_j \qquad (27)$$

Where ψ_j is the complex eigenvector of **A** corresponding to the eigenvalue λ_j

4. Verification through numerical simulation

In order to verify the validity of the proposed method in this paper, a three degree of freedom vibration structure system as following (figure 1).

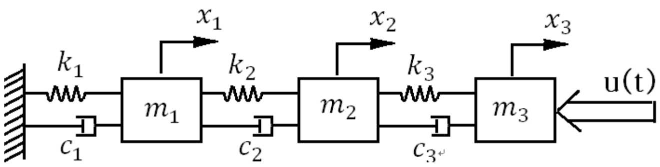

Fig. 1. 3 degrees of freedom vibration structure system

In figure 1, external excitation is applied through point m_3 and is expressed as u(t).

The physical parameters in the given structure vibration system are set as follows.

m_1=10kg, m_2=15kg, m_3=20kg

k_1=10kg, k_2=15kg, k_3=20kg

c_1= 3n/s, c_2 = 5n/s, c_3=10n/s

As the external excitation u(t), we used triangluar form signal as shown in figure 2.

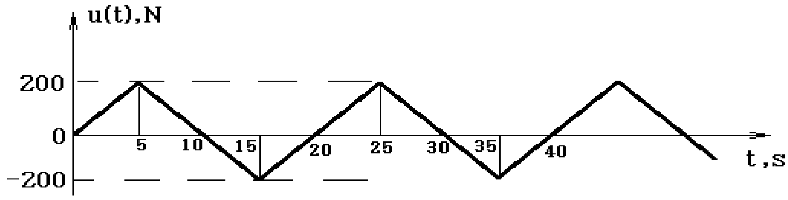

Fig. 2. External excitation diagram according to time

Random noise with a covariance corresponding to 10% of nominal values was added in viscous coefficiances c1, c2, c3. At the same time,a random noise with a variance corresponding to 5% of the excitation maximum value was added in excitation. We added random noise corresponding with the measurement noise level of low cost acceleration sensors to measurement values.

From table 1, it can be seen that relative error between theorical values and identification results is less than 15% in damping ratio and less than 10% in the eigenfrequences and mode shape. That is, modal parameters were well identificated even in the presence of process noise and measurement noise.

From now on, the validity of the method proposed in this paper was proved.

Comparison of theorical value and identification result Table 1

Table 1. Shows the results of the identification of the modal parameters obtained by using the Kalman filter algorithm

Normal frequence, Hz			
	theory	identification	relative error,%
1th	2.1014	1.9901	5.4785
2th	7.9821	8.5580	7.2147
3th	11.9635	11.2068	6.3254
Damping ratio,%			
	theory	identification	relative error,%
1th	6.5	5.89	9.5
2th	0.4	0.443	10.8
3th	0.4	0.459	14.8
Mode shape			
	theory	identification	relative error,%
1th	(0.5878 0.8875 1.000)	(0.6063 0.8354 1.000)	(3.1457 5.8741 0)
2th	(1.000 0.3070 -0.4982)	(0 0.2875 -0.4796)	(0 6.3651 3.7415)
3th	(-0.8693 1.000 -0.4102)	(-0.9436 0 -0.3818)	(8.5417 0 6.9214)

5. Conclusion

In this paper, we proposed the methodology to indentify modal parameters of a structure vibration system by using kalman filter algorithm, which becomes one of the powerful methods of modern control theory.

By using the kalman filter algorithm, it is possible to identify modal parameter optimally even in the presence of process noise and measurement noise exists.

The performance and validity of the proposed methodology was verificated through simulation application.

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