Dependence of optimal solution and optimal value on constraints set

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Зависимость оптимального решения и оптимального значения от множества ограничений

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Abstract: *any issue in the present case, the mathematical models of learning are also different according to different approaches. Understanding how the optimal solution of the issues of this type of research is important. The optimal solution is to change the assessment of the limitations affecting the majority of the changes in the solution.*
Аннотация: *в статье рассматривается зависимость оптимального решения и оптимального значения от множества ограничений. Даются постановка и применение задачи оптимального управления, связанные с изменением множества ограничений по параметру.*

Keywords: *optimal solution, constraints set, objective function.*
Ключевые слова: *оптимальное решение, множества ограничений, целевая функция.*

Study of optimal plan on resources or in other words on constraints set is a very urgent problem. This is very important from the point of view of influence of change of constraints set a change of the solution in the solution. But intestigation of this dependence because of complexity and strong linearity of its character, is connected with some difficulties. There may happen so that alternation of constrainds after some state generally doesn't change the solution. For example, of a strong convex functionbntakes its minimum value in all of the space at the point x_0, then independent on the change of the set containing the point x_0, this doesn't reduse to afternation of the optimal point, the point x_0. Dependence of optimal solution on the objective function is an dependence. So, negligible change of a function reduse to sharp change of optimal solution, or optimal value.

For example, of the minimum value of the function $f_1(x) = 0.001x^2$ in all of the space is zero, then minimal value of the function $f_2(x) = -0.001x^2$ is infinity.

All what has been said showes complicated character of dependence of optimal value on constraints and objective function. But for some class of problem, especially for economical problems, this dependence some confirmities are refained under this dependence and it because applicable.

Assume that it is regular to minimize the function $f(x)$ in the set D from any fonite-dimensional space R^n. In other wodrs, we are interested in the following problem

$$f(x) \to \min, \quad x \in D \qquad (1)$$

It is obvious that the optimal solution $x_0 \in D$ and optimal value $f(x_0)$ is depended on the objective function and the set D. That's why, the optimality condition that characterizes the solution should be directly dependend on the set and objective function.

Assume that the set D in a convex set. Then it is clear that it is uniquely determined by its own support [2, 3]

$$P_D(x) = \sup_{l \in D}(l, x), \quad x \in R^n \qquad (2)$$

and this function is convex, positive homogeneous and continuous. For such an arbitrary, convex, positive homogeneous and continuous function $P(x)$ there exists a set $D \in M$ such that $P(x) = P_D(x)$. At the zero point the set D is determined is a sub differential of the function $P(x) = P_D(x)$

$$D = \partial P(0) = \{l \in R^r : P(x) \geq (l, x), x \in R^n\}.$$

So, it would be natural to write the set D characterizity the objective function $f(x)$ of optimality condition and constaints in the termin of a support function.

Theorem 1. Suppose that the function $f(x)$ is continuously differentiable in the set $D \subset R^n$ and $x_0 \in D$ in the solution of problem (1) then the following condition is satisfied

$$(f'(x_0), x_0) + P_D(-f'(x_0)) = 0 \qquad (3)$$

If the function $f(x)$ is convex in the set D, then condition (1) is suffiwent for the point $x_0 \in D$ to be the solution of problem (1).

Proof. Assume that $x_0 \in D$ is the solution of problem (1). Then it is[1] known that for arbitrary $x \in D, (f'(x_0), x - x_0) \geq 0$.

Hence we get $(f'(x_0), x_0) \leq -(-f'(x_0), x), \quad \forall x \in D$.

It is obvious that

$$(f'(x_0), x_0) = \inf_{x \in D}\left[-(-f'(x_0), x)\right] = \sup_{x \in D}(-f'(x_0), x).$$

By definition of support function (2), $P_D(-f'(x_0)) = \sup_{l \in D}(-f'(x_0), l)$

from this relation we get $(f'(x_0)x_0) = -P_A(-f'(x_0))$ i.e. relation (3) is satisfied.

Now, assume that the function $f(x)$ is convex on the set D. Then from condition (3) we get

$$0 = (f'(x_0), x_0)) - \inf_{l \in A}(f'(x_0), l) \geq (f'(x_0), x_0) - (f'(x_0), x),$$

or

$$(f'(x_0), x) \geq (f'(x_0), x_0), \ \forall x \in D.$$

Hence, by using [1] the known result, we see that the point $x_0 \in D$ is the solution of problem (1).

For example, majority *D* is following a single ball:

$$D = \left\{x \in R^n : \|1\| \leq 1\right\}.$$

Here $\| \ \|$ is norm of R^n. It stands support function as a single ball $P_D(x) = \|x\|$, given that the point $x_0 \in D$ of minimum functions $f(x)$ for a variety of D, (3) we can write the condition as follows:

$$\left(f'(x_0), x_0\right) + \|f'(x_0)\| = 0.$$

Birdəyisənli case, this condition is following:

$$f'(x_0) \cdot x_0 + |f'(x_0)| = 0$$

In other words, either $f'(x_0) = 0$, or $x_0 = \begin{cases} -1, \ f'(x_0) > 0 \\ +1, \ f'(x_0) < 0 \end{cases}$.

Now, for example, function $f(x)$ the linear function of the abundance of D, ie,

$$f(x) = c_1x_1 + c_2x_2 + \ldots + c_nx_n = (c, x). \qquad (4)$$

Here, $c = (c_1, c_2, \ldots c_n)$. In this case $f'(x) = c$ and therefore (3) termwill receive the optimal solution for $x_0 \in D$:

$$(c, x) + P_A(-c) = 0.$$

Given that the minimum rate (c, x_0) of here, we can give the following results.

Conclusion.For example, a point $x_0 \in D$ the line (4) function provides a minimum of D variety.For the minimum rate

$$f_{\min} = (c, x_0) = -P_D(-c) \qquad (5)$$

As we see an excess of the minimum rate in the D and depends on the vector. $c = (c_1, c_2, \ldots c_n)$.

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